Heat Transfer Write-Up

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Introduction

This is a write-up on behalf of my heat transfer course (MAE 3240), for which I go through the derivation of a problem, then explain the things I learned as a result of the problem set.

Problem Statement

The emissivity of a room material is $\epsilon = 0.18$. The absorptivity of this material for solar irradiation is $\alpha = 0.76$.

1. Consider a day with solar irradiation

   $$G_s = 750\,\mathrm{W\,m^{-2}}.$$

   The surrounding ambient temperature is

   $$T_\infty = 300\,\mathrm{K},$$

   and the surrounding surface temperature is

   $$T_{\mathrm{surr}} = 300\,\mathrm{K}.$$

   The convective heat transfer coefficient of the air flow is

   $$h = 10\,\mathrm{W\,m^{-2}\,K^{-1}}.$$

   Calculate the temperature of the roof. Assume the bottom surface can be treated as a gray body exposed to irradiation from the surrounding environment.

   Would a neighboring cat be comfortable walking on a roof constructed from this material?

2. Based on the calculation in part (1), what is the dominant heat transfer mode?

Solution

First, I began by sketching the setup with the given constants and identifying the appropriate system.

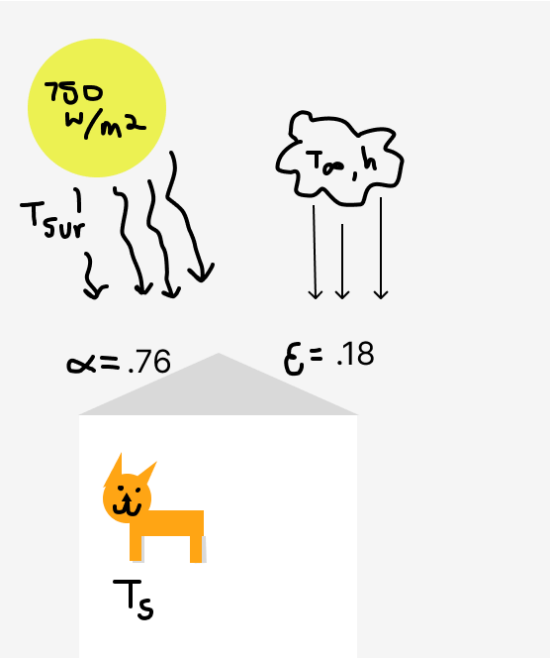


Figure 1: Sketch illustrating the problem setup

Next, the following assumptions were made:

- The system is in steady state.
- The roof is thermally insulated underneath.
- The roof behaves as a gray body with respect to the surrounding irradiation.

Energy Balance

An energy balance is performed on the control volume at the roof surface:

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

For a gray body exchanging radiation with its surroundings, Kirchhoff's law allows us to approximate the absorptivity and emissivity as equal for thermal radiation:

$$\alpha \approx \epsilon = 0.18$$

The incoming energy consists of:

- Solar irradiation
- Irradiation from the surroundings

Thus,

$$\dot{E}_{in} = \alpha_s G_s + \alpha G_{\text{surr}}$$

The surrounding irradiation can be expressed using the Stefan–Boltzmann law:

$$G_{\text{surr}} = \sigma T_{\text{surr}}^4$$

where

$$\sigma = 5.67 \times 10^{-8}\,\text{W}\,\text{m}^{-2}\,\text{K}^{-4}$$

Substituting values:

$$\dot{E}_{in} = 0.76(750) + 0.18(5.67 \times 10^{-8})(300^4)$$

$$\dot{E}_{in} \approx 652.7\,\text{W}\,\text{m}^{-2}$$

As for the modes of energy transfer out, they consist of convection and thermal radiation emitted from the roof surface. Thus,

$$\dot{E}_{out} = q''_{\text{conv}} + q''_{\text{rad}}$$

The convection and radiation terms are given by

$$q''_{\text{conv}} = h(T_s - T_\infty)$$

$$q''_{\text{rad}} = \epsilon\sigma T_s^4$$

Thus,

$$\dot{E}_{out} = h(T_s - T_\infty) + \epsilon\sigma T_s^4$$

Substituting the given values,

$$\dot{E}_{out} = 10(T_s - 300) + 0.18(5.67 \times 10^{-8})T_s^4$$

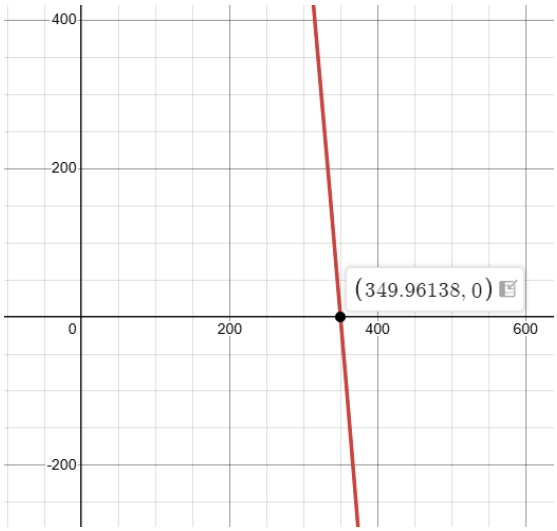


Figure 2: Solution for roof surface temperature

$$\dot{E}_{out} = 10(T_s - 300) + 0.18(5.67 \times 10^{-8})T_s^4 \quad [\mathrm{W\,m^{-2}}]$$

Setting the energy rate in equal to the energy rate out provides an equation to solve for the surface temperature of the roof:

Thus, the temperature of the roof is

$$T_s = 349.96\,\mathrm{K}$$

which is approximately

$$T_s \approx 77\,^{\circ}\mathrm{C}.$$

This temperature is far too hot for a cat to walk on comfortably, given that the average body temperature of a cat is approximately 38 °C to 40 °C.

Dominant Heat Transfer Mode

To determine the dominant mode of heat transfer, the heat flux due to convection and radiation are calculated.

- **Convection**

$$q''_{\mathrm{conv}} = h(T_s - T_\infty)$$

Substituting values,

$$q''_{\mathrm{conv}} = 10(350 - 300) = 500\,\mathrm{W\,m^{-2}}$$

- **Radiation**

$$q''_{\text{rad}} = \epsilon \sigma T_s^4$$

Substituting values,

$$q''_{\text{rad}} = 0.18 \times (5.67 \times 10^{-8}) \times (350^4)$$

$$q''_{\text{rad}} = 153\,\text{W}\,\text{m}^{-2}$$

Since

$$q''_{\text{conv}} > q''_{\text{rad}},$$

convection is the dominant mode of heat transfer in this case.

Discussion

As a whole, this question and the other questions involved on the first homework assignment for Heat Transfer (MAE 3240) were very beneficial to my development in understanding modes of heat transfer, as well as day-to-day uses of heat-transfer and its underlying properties.

For the first question, I was tasked with comparing the heat transfer modes for a satellite in different regions of the atmosphere. I was able to convince myself that radiation is the most important mode of heat transfer when outside of the atmosphere because there is no medium for which the other two heat-transfer modes can function (aside from a small amount of conduction within the temperature gradients of the satellite itself). Then, during re-entry, the main mode is convection given the high-speeds of the medium through which the satellite is traveling. Conduction occurs, too, however, it is much less significant given the speed and the resulting convection is many orders of magnitude larger. As for the second question, I was tasked with understanding the heat-transfer means on a computer-chip, which has a great deal of relevance in regards to robotic design and general computation. I used Fourier's law for conduction through the chip on-to the surface of the chip where a convection mode most efficiently transferred heat away from the chip. I learned a lot about the role of fans in heat-transfer as they increase the convective heat transfer coefficient by many orders of magnitude, thus the computer chip's steady state temperature is much lower than any arbitrarily designated cut-off.

As for the final question, I learned the importance of radiation outside of an atmospheric implication, and I learned to include more relevant estimations to overcome a lack of information (such as making the grey-body approximation), as well as the many sources of radiation that I hadn't considered (aside from the sun).